DYNAMICS OF THE FORMATION OF BUBBLES IN A VIBRATING CAPILLARY

I. S. Grachev, D. T. Kokorev, and V. F. Yudaev

UDC 532.529.6

The instant of separation of a gas bubble from a liquid occupying a vibrating capillary at Reynolds numbers of Re < 100 is considered. The dimensions of the bubble are determined analytically in relation to the amplitude and frequency of vibration of the capillary. The results are compared with experiment. Three basic conditions of bubble formation are distinguished.

The intensification of heat- and mass-transfer processes in liquid—liquid and liquid—gas systems may be achieved by increasing the surface of contact of the reacting phases, and also by establishing a favourable hydrodynamic situation in the zone of diffusive transfer — this may be done by suitable apparatus design. For example, in the case of processes in a liquid—gas system it is desirable to ensure the smallest possible bubble size in a capillary of specified dimensions, subject to an adequate rate of flow [1]. In order to solve this problem, we propose dispersing gases and liquids in liquids, and liquids in gases (i.e., "granulating" the liquids), by means of a vibrating capillary.

In order to simplify the analytical determination of the size of the bubble escaping from the vibrating capillary, we shall introduce an approximation by treating the bubble as rigid and spherical.

During the to-and-fro motion of the bubble in the vertical direction, the following forces act upon it: the Archimedes lifting force $F_A = \Delta\rho gV$; the retaining capillary force $F_c = 2\pi\sigma R$; the Stokes force $F_S = 6\pi\nu\rho a U$; the force of inertia $F_b = M(dU/dt)$ arising as a result of the nonuniform to-and-fro motion of the bubble in the liquid. The first two forces depend solely on the dimensions of the neck and the bubble; they are determined as in the case of a static capillary. The last two forces are due to the vibration of the capillary. The forces F_S and F_b acting on a bubble the center of which oscillates harmonically

$$x = A\sin\omega t, \tag{1}$$

may be expressed thus

$$F_S = 6\pi\nu\rho a A\omega\cos\omega t, \tag{2}$$

$$F_b = -MA\omega^2\sin\omega t. \tag{3}$$

It follows that the extremal value of the sum of the forces

$$\frac{d}{dt}(6\pi\nu\rho a A\omega\cos\omega t - MA\omega^2\sin\omega t) = 0 \tag{4}$$

is determined by the phase of the oscillation

$$\omega t_e = \operatorname{arctg}\left(-\frac{M\omega}{6\pi\nu\rho a}\right), \tag{5}$$

where, from the conditions governing the detachment of the bubble, we must take the signs of the trigonometric functions thus:

$$\sin\omega t_e = -\frac{M\omega}{\sqrt{(6\pi\nu\rho a)^2 + (M\omega)^2}},$$

Moscow Institute of Chemical Engineering. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 3, pp. 424-429, September, 1973. Original article submitted March 28, 1973.

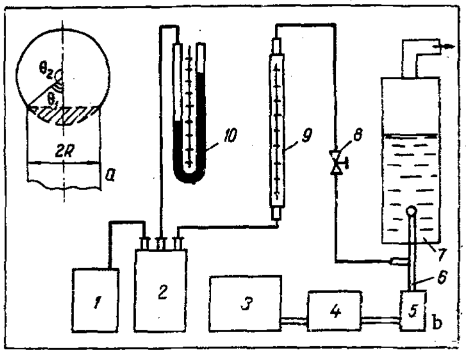

Fig. 1. Diagram to illustrate the derivation of the basic relations (a) and form of the experimental apparatus (b): 1) compressor; 2) receiver; 3) sound generator; 4) amplifier; 5) electrodynamic converter; 6) capillary; 7) column; 8) diaphragm; 9) flowmeter; 10) manometer.

$$\cos\omega t_e = \frac{6\pi\nu\rho a}{\sqrt{(6\pi\nu\rho a)^2 + (M\omega)^2}}, \tag{6}$$

so that the phase ωt_e may correspond to the third quarter of the period, when the forces (2) and (3) are directed upward. The sign of the second derivative of the sum (2) and (3) in the phase (6) determines the character of the extremal point

$$\frac{d^2}{dt^2}(F_S + F_b)_{t=t_e} = A\omega^2 \frac{(6\pi\nu\rho a)^2 - (M\omega)^2}{\sqrt{(6\pi\nu\rho a)^2 + (M\omega)^2}}. \tag{7}$$

For the condition

$$\frac{M\omega}{6\pi\nu\rho a} > 1 \tag{8}$$

in the extremal phase, the sum of the forces is at a maximum, since the second derivative (7) is less than zero. Otherwise the sum of the forces (2) and (3) is a minimum.

On satisfying Eq. (8), the inequality $F_S < F_b$ in the extremal phase of the vibration (6) is strengthened by the fact that we now also have the inequality

$$-\sin\omega t_e > \cos\omega t_e \tag{9}$$

and the middle section of the gas bubble diminishes during its downward motion in the trace of the capillary, which leads to a reduction in the frontal resistance F_S. Without allowing for this latter factor, we may determine the ratio of the maximum sum of the forces $(F_S + F_b)_{max}$ to the maximum value of the force $(F_b)_{max}$ thus

$$\frac{(F_S + F_b)_{max}}{(F_b)_{max}} = \sqrt{1 + \left(\frac{6\pi\rho\nu a}{M\omega}\right)^2}. \tag{10}$$

Hence, for sufficiently large forces F_S and F_b in low-viscosity liquids, for which the condition

$$\left(\frac{6\pi\rho\nu a}{M\omega}\right)^2 \ll 1, \tag{11}$$

is satisfied, we may assume that a varying force F_b acts on the bubble, and this force determines the instant of detachment of the bubble from the capillary at the upper point. For low frequencies the alternating forces are small and the detachment of the bubble is determined by the forces F_A and F_c.

On satisfying Eq. (11), the forces acting on the bubble at the upper point are determined thus*

$$F_A + F_b = F_c. \tag{12}$$

The augmented mass M of the translational, nonuniform motion of the sphere in the liquid referred to in Eq. (3) may be found [2] from the well-known velocity potential

$$\varphi = \frac{1}{2} U \frac{a^3}{r^2} \cos\theta \tag{13}$$

and the definition of the kinetic energy of the liquid W

$$2W = MU^2 = -\rho \int_0^{2\pi} \int_{\arccos\sqrt{1-C^2}}^{\pi} \varphi \frac{\partial\varphi}{\partial r}\, ds \tag{14}$$

Here the limits of integration are determined from Fig. 1a, allowing for the direction of motion of the gas bubble. It follows from (13) and (14) that

$$M = \frac{\pi\rho a^3}{3}\left[1 + \sqrt{(1 - C^2)^3}\right]. \tag{15}$$

*In general when $(F_S + F_b)_{max} + F_A = F_c$ we obtain an equation of the sixth degree for the radius of the bubble. An analysis of the equation may be required for the dispersion of gases and liquids in-liquids and the granulation of liquids (melts) in a gas.

If we allow for the finite dimensions of the capillary (neck of the bubble) we have

$$F_A = \Delta\rho g (V_{sp} - V_{seg}). \tag{16}$$

Calculations show (Fig. 1a) that

$$F_A = \frac{1}{6}\pi\Delta\rho g a^3\left[1 + \left(7 - \frac{C^2}{2}\right)\sqrt{1 - C^2}\right]. \tag{17}$$

Allowing for the latter equation and remembering that at the point under consideration [(3) and (15)]

$$F_b = \frac{1}{3}\pi a^3\rho\left[1 + \sqrt{(1 - C^2)^3}\right] A\omega^2, \tag{18}$$

Eq. (12) takes the form ($\Delta\rho \approx \rho$)

$$\frac{1}{6}\pi a^3\rho g\left[1 + \frac{3}{2}C^2 + \left(7 - \frac{C^2}{2}\right)\sqrt{1 - C^2} + 2(1 + \sqrt{(1 - C^2)^3}\frac{A\omega^2}{2g}\right] = 2\pi\sigma R \tag{19}$$

In particular, for a static capillary [1]

$$a_0^3 = \frac{3}{2}\frac{R\sigma}{\rho g}. \tag{20}$$

From (19) and (20) we obtain

$$\frac{a_0^3}{a^3} = \frac{V_0}{V} = \frac{1}{8}\left[1 + \frac{3}{2}C^2 + \left(7 - \frac{C^2}{2}\right)\sqrt{1 - C^2}\right] + \frac{1}{2}\left[1 + \sqrt{(1 - C^2)^3}\right]\frac{A\omega^2}{2g} \tag{21}$$

or

$$\frac{V_0}{V} = K_1(C^2) + K_2(C^2)\frac{A\omega^2}{2g} \tag{22}$$

Here $K_1(C^2)$ is a function of the volumetric deformation [volume deformation factor (16)], while $K_2(C^2)$ is a function of the form of the bubble surface [form factor (14)]. For a rigid spherical particle K_1 and K_2 may be found from Eq. (21). For any other form of the bubble, allowance must be made for (14) and (16). The argument C is different for K_1 and K_2 if the bubble is not spherical.

In order to study the formation of a bubble on a vibrating capillary experimentally, we made the apparatus illustrated in Fig. 1b. The volume of the bubble was determined as the mean of several bubbles. The experiments were made with a capillary of radius 1.6 mm, and amplitude and frequency f of the vibrations being regulated by means of a UM-50A amplifier and ZG-11 generator, between 20 and 250 Hz; the vibrator was an electrodynamic converter of the 100GRD-III-I type. The experiments were carried out in the distilled water—air system. The pressure in the receiver was kept constant; the flow of air was regulated with a diaphragm.

The results of our experiments to determine the size of the bubble as a function of the relative acceleration, defined by the complex $A\omega^2/2g$, are presented in Fig. 2. From Fig. 2 we may draw the following conclusions.

1. For low frequencies and considerable amplitudes (curves 1 and 2), the coefficients K_2 are greater than unity, and K_1 of the order of unity.

2. For all frequencies and small amplitudes (initial parts of the curves) $K_1 = 1$.

3. For the frequencies studied, K_2 increases with the amplitude while K_1 becomes negative. For certain values of the complex $A\omega^2/2g$ microscopic bubbles are formed in a transient manner (asymptotes of curves 3-5).

4. For low (curves 1 and 2) and high (curves 6-8) frequencies we were never able to reach the amplitudes at which the transient dispersion of the gas through the macroscopic capillary began.

5. Both from the point of view of energy consumption and from that of the action of the vibrating capillary ion the dispersion of gas in the liquid, the least effective frequencies for the capillary under

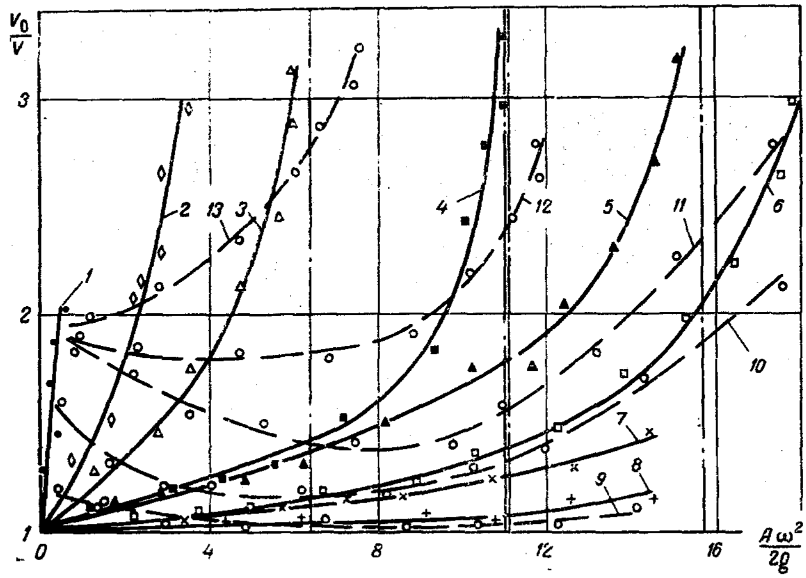

Fig. 2. Experimental dependence of the relative volume of the bubble on the vibrational acceleration of the capillary. For a constant cyclical frequency (Hz) of the vibrations of the capillary (continuous curves): 1) 20 Hz; 2) 40; 3) 60; 4) 80; 5) 100; 6) 140; 7) 180; 8) 250; for a constant amplitude (mm) (broken curves): 9) 0.1; 10) 0.3; 11) 0.5; 12) 0.7; 13) 0.9.

consideration are those above 100 Hz. At high frequencies the model of a rigid spherical bubble does not correspond to experimental facts.

6. The dispersion of the gas in the liquid in a vibrating capillary depends not only on the complex $A\omega^2/2g$ but also on the amplitude of the vibrations of the capillary, which (other conditions being equal) affect the conditions relating to the rigid spherical bubble. For low frequencies and a large amplitude, the bubble has the form of an elliptical cap directed with its vertex toward the capillary, while the major semiaxis coincides with the direction of motion. In this case the Archimedes force does not change (coefficient K_1) while the augmented mass increases sharply (coefficient K_2, equal to the slope of the tangents to the curves 1 and 2). For high frequencies and a small amplitude, the bubble has a spherical shape, and the coefficients K_1 and K_2 are similar to the calculated values of Eqs. (14) and (16) (curves 7 and 8). For amplitudes of 0.5-1.0 mm and frequencies of 60-100 Hz, the shape of the bubble is unstable; it disperses into micro-bubbles (asymptotes of curves 3-5).

From a consideration of the curves and also visual observations, we may distinguish three main conditions for the formation of bubbles: 1) that of a transient bubble shape for low frequencies and large amplitudes; 2) that of a fan-like atomization of the gas for fairly large amplitudes and moderate frequencies; 3) that of a "steady" shape of the bubble, almost spherical for high frequencies and small amplitudes, when there is little relative motion of the bubble and the liquid, so that the forces (2) and (3) are also small. This is the case in which the action of capillary vibration on the formation of the bubble is least effective.

A quantitative estimation of these three distinct conditions of bubble formation will probably involve the dimensions of the capillary, the viscosity and surface tension between the dispersed and dispersing phases, the pressure and rate of flow of the dispersing phase, and so on.

NOTATION

F_A	is the Archimedes lifting force;
$\Delta\rho$	is the difference between the densities of the liquid and the gas;
a	is the radius of the bubble;
g	is the gravitational acceleration;
F_c	is the retaining capillary force;
R	is the radius of capillary (neck of the bubble at the instant of detachment);
σ	is the surface tension at the interface;

ds	is the element of surface area on the bubble in a spherical coordinate system;
F_S	is the Stokes force;
ν	is the kinematic viscosity of the liquid;
ρ	is the density of the liquid;
F_n	is the inertial force;
M	is the augmented mass;
U	is the variable velocity of the to-and-fro motion of the bubble;
t	is the time;
r, θ	are the spherical coordinates;
x	is the displacement of the center of the bubble and the capillary from the static equilibrium position;
A	is the amplitude of vibration of the capillary;
f, $\omega = 2\pi f$	are the cyclical and angular frequencies of the vibration of the capillary;
φ	is the velocity potential of the liquid;
W	is the kinetic energy of the liquid;
V	is the volume of the bubble obtained on the vibrating capillary;
V_0, a_0	are the volume and radius of the bubble obtained on the static capillary;
V_{sp}, V_{seg}	are the volumes of the sphere and segment; $C = R/a$; $Re = Ua/\nu = A\omega a/\nu$.

LITERATURE CITED

1. S. L. Saw, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1971).
2. H. Lamb, Hydrodynamics [Russian translation], Gostekhizdat, Moscow (1947).